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Report on the scientific achievement presented by Dr. Piotr Pokora for his habilitation procedure.

For his habilitation procedure Dr. Piotr Pokora has presented 7 papers, written either by himself or in collaboration, as his major scientific achievement. These papers are mainly in Algebraic Geometry, more precisely in the study of algebraic surfaces and algebraic curves on these surfaces. This is a classical subfield of Algebraic Geometry, still full of unsolved conjectures and open questions.

Instead of describing in detail the content of these 7 papers, that is done nicely in Dr. Pokora's self-report, I will present my comments on each of these papers and summarize their contribution.

The first one of presented papers "On the boundedness of the denominators in the Zariski decomposition on surfaces" published in Journal für die reine und angewandte Mathematik and written in collaboration with T. Bauer and D. Schmitz, contains a surprising, and even intriguing, result relating two major conjectures : Zariski denominator conjecture and the bounded negati-

vity conjecture. Both conjectures describe numerical invariants related to the divisors (i.e. curves) on smooth algebraic surfaces. The main result of this paper is then that, for a smooth projective surface X over an algebraically closed field, the following two conditions are equivalent : X has bounded Zariski denominators and X has bounded negativity. Moreover, the authors provide partial relations between the numerical invariants involved. This is clearly an important contribution.

The bounded negativity conjecture states that every smooth complex projective surface X has bounded negativity, i.e., the self-intersection of every reduced and irreducible curve $C \subset X$ (or equivalently the self-intersection of every reduced divisor D) is bounded from below. In the papers "Bounded negativity, Harbourne constants and transversal arrangements of curves" published in Ann. Inst. Fourier and written in collaboration with X. Roulleau and T. Szemberg, and "On the local negativity of surfaces with numerically trivial canonical class" published in Rendiconti Lincei and written in collaboration with R. Laface, P. Pokora obtains several new bounds for the self-intersection of some particular reduced divisors on the blow-ups of smooth surfaces. In particular he gets the bounds for the local Harbourne invariant and generalizes a Hirzebruch-type inquality for rational curves arrangements. The main tools include the Bogomolov-Miyaoka-Yau inequality. These results constitute an important contribution towards understanding the bounded negativity conjecture and the related phenomena.

In the paper "Harbourne constants under ramified morphisms", published in Results in Mathematics and written by P. Pokora in collaboration with J. Roé, the authors introduce the notion of Harbourne constant at a multicluster of infinitely near points, extending the definition of Harbourne index to arbitrary curves on smooth surfaces. Among other things, they provide an example of a configuration of curves in $\mathbb{P}^2$ with Harbourne index lower than Wiman's configuration of lines (which previously held the record).

In the survey article "Hirzebruch-type inequalities viewed as tools in combinatorics", published in Electronic Journal of Combinatorics, P. Pokora provides an introduction, algebro-topological in nature, to Hirzebuch-type inequalities for plane curve arrangements in the complex projective plane. This paper also contains new results mainly related to the Weak Dirac Conjecture.

A part of P. Pokora's work is devoted to the theory of conic-line arrangements in particular in the context of the free divisors (in the sense of K. Saito) and almost free divisors. A part of this work is obtained in collaboration with A. Dimca, one of the world's leading experts in this area. In the paper "On conic-line arrangements with nodes, tacnodes, and ordinary triple points" published in Journal of Algebraic Combinatorics, they obtained a complete classification of such arrangements. As a corollary they show that Numerical Terao's Conjecture holds for such arrangements. In the paper "Conic-line ar-

rangements in the complex projective plane" written in colaboration with T. Szemberg and published in Discrete and Computational Geometry, P. Pokora shows a de Bruijn–Erdös type inequality and Hirzebruch-type inequality for a certain class of conic-line arrangements having ordinary singularities.

The presented papers are published in good and very good, though not of the first rank, journals. Altogher, after Math.Sci.Net, Dr Pokora is the author of thirty four research papers that contain many more important results. We note a succeful collaboration of Dr Pokora with several world leading experts including T. Bauer, A. Dimca, B. Harbourne, and X. Roulleau, and several visits and longer stays in renowned centers. Dr Pokora has supervised eight bachelor's theses and two master's theses, and is currently supervising four master's theses and two doctoral students.

Conclusion. The scientific achievements presented by Dr. Pokora for his habilitation contain several non-trivial and original results that contributed significantly to the study of curves on algebraic surfaces, an important area of Algebraic Geometry. Therefore, I am favorable to award to Dr. Piotr Pokora the degree of "doktor habilitowany".

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