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Report on the Habilitation application of Dr. Justyna Szpond

The candidate works in Algebraic Geometry, more precisely in enumerative and quantitative aspects of hyperplane arrangements in projective spaces (over various fields). This is a classical subject, it goes back even to antiquity (Pappus), so needles to say that any progress requires an original idea. Nowadays it is a very active area linked to Combinatorics.

In the application the candidate listed 26 papers published in peer-reviewed journals and 5 chapters in monographs. She has chosen 7 recent papers for detailed presentation, in 4 of them she is the sole author.

In a very clear exposition of the results in those 7 papers, the candidate describes very efficiently the state of art in the area and her contributions. There are two main directions of her research: the containment problem and the existence of unexpected hypersurfaces.

The containment problem asks for a sufficient condition for inclusion of the m-rd symbolic power of I in the r-rd power of I, where I is a homogenous ideal in the ring of polynomials in N+1 variables over a field K. Around year 2000 Ein, Lazarsfeld, Smith and independently Hochster, Hunke proved that it is enough to assume that m is greater or equal than Nr. (Both papers were published in Inventiones). This is probably not a sharp condition, it is conjectured that it is enough that m is greater or equal than Nr -(N-1). For n=r=2 it gives a conjecture that the 3-rd symbolic power of I is contained in the 2-nd power of I. Actually this conjecture was disproved, first over complex numbers and also over fields of positive characteristic. The candidate was involved in a construction of an example over reals (based on Boroczky arrangement). All these examples have some nontrivial symmetry, which cannot be realized over the rationals. In a joint work with Lampa-Baczynska she found a nice

perturbation of Boroczky arrangements. As a consequence they obtained a counter example to the containment conjecture over the rationals. The candidate gave also an interesting explicit study of perturbations of some Boroczky arrangements. This is a very beautiful construction in real algebraic geometry.

The second topic presented in the application is the existence of unexpected hypersurfaces. Its purpose is to describe those projective varieties for which there are polynomials that vanish on the given variety and vanish on a finite set of general points with prescribed multiplicity. These polynomials are called unexpected if the number of conditions describing the data is bigger than the dimension of the polynomials, for given degree d. This problem has attracted the attention of numerous mathematicians over the past three centuries and remains one of the central problems in Algebraic Geometry. It has strong relations and ramifications to interpolation theory, combinatorics, discrete mathematics and many others.
The main contribution of the candidate to this subject is: a discovery of a link between dual sets to Fermat-like arrangements for which unexpected hypersurfaces exist, the role of higher osculating spaces for the existence of unexpected hypersurfaces. The last part is related to Laplace equations.

Dr. Szpond obtained several significant results on the classical problems in Algebraic Geometry, needles to say that this a very difficult matter. She presented a very convincing research program based on her achievements.

She is systematically invited as a speaker to international meetings, workshops and seminars in the subject. This proves the recognition by the community.

Dr. Szpond is the principal investigator in a research grant and a co-investigator in 3 others. She organized 13 international meetings, received several prizes from her university for the research activity. The candidate is a co-adviser of one Phd-Theses and has strong activity in popularization of mathematics.

In my opinion Dr. Szpond has clearly the maturity of an independent researcher, accordingly I strongly support her application for Habilitation in Mathematics.

Professor Krzysztof Kurdyka