To the Faculty of Mathematics and Computer Science

of the University of Łódź

Report for the Habilitation Procedure of Dr. Justyna Szpond

Dr. J. Szpond presents a research activity in the field of Algebraic Geometry.

Following the scientific production of the candidate, the main interests and results of Dr. Szpond are related to configuration of points, possibly with multiplicities, and their interpolation properties with respect to homogeneous polynomials, in projective spaces.

The candidate presents 7 papers, 4 of which in collaboration, printed in a period ranging from 2017 to 2020. 5 papers appeared in international journal of good reputation. 2 papers are published in a series of proceedings. The MathSciNet index (MCQ) of the journals in which the candidate published the papers ranges from 0,56 to 0,82, standard values for mathematical journals of international level. According to the Scopus archive, the average number of citations for the presented papers is 4,25, a good value for publications in Algebra and Geometry.

The main scientific results obtained by the candidate concern mainly the following two problems, both linked to polynomial interpolation:

- The containement problem, whose target is the study of the relation between symbolic and algebraic powers of ideals of projective varieties. From a geometrical point of view, the problem can be described as the relations between imposing to polynomials the vanish along a variety X with multiplicity m versus raising to the m-th power polynomials vanishing simply along X. The containement problem determines several useful invariants for algebraic varieties, expecially in the case where X is a configurationunion of linear subspaces. The main achievements of the candidate is the construction of examples of configurations of linear speces, even over the field Q of rationals, for which the containment does not satisfy the expected behviour, and in particular the third symbolic power of an ideal I does not lie in the second power of I.

- The unexpected hypersurface problem, whose target is the construction and the description of varieties X for which one can find unexpected polynomials f of certain degrees vanishing along X and vanishing at general sets of points with prescribed multiplicities. These polynomials f are unexpected when the number of conditions imposed by X and the multiple points exceed the dimension of the space of polynomials. Special varieties X for which unexpected hypersurfaces exist are of great interest for interpolation theory, expecially in the case where X is an arrangement of linear subspaces (including the case where X is a finite set). Such configurations have rich combinatorial structures, and provide important examples for discrete mathematics and geometry.The main results achieved by the candidate is the description of sets of points in the plane, which are dual to special arrangements of lines called ‘Fermat arrangements’, for which unexpected hypersurfaces exist. The candidate also found a relationship between sets X with unexpected hypersurfaces

and projective varieties whose osculating space have a degenerate behavior. Such varieties X are special cases of varieties which satisfy Laplace equations, a type of differential equations deeply studied in classical and modern literature.

The achievements of the candidate certainly provide a significant contribution to the development of Algebraic Geometry, and its connections with Combinatorics. The researches also suggest several directions for future promising investigations.

The candidate illustrated the scientific achievements in conferences and seminars in Poland, Germany, Sweden, and Italy. Dr. Szpond received two rector's prizes for scientific activities and two mentions in competitions for best papers in Mathematics.

Dr. Szpond presents an activity of organizer for one workshop in EuroMath 2018 in Cracow, and an activity of co-organizer for 13 scientific meetings located in Poland and Germany. She also presents the activity of principal investigator in one research fund, and activities of co-investigator in 3 research funds. The candidate is Managing Editor for the Annales Universitatis Paedagogicae Cracoviensis Studia Mathematica.

Dr. Szpond educational activities include the teaching of several courses at undergraduate and PhD level. The candidate is co-advisor of one PhD Thesis, and co-author of one handbook for students.

The candidate organized events and coordinated funds for activities of popularization of Mathematics.

Summarizing, the researches of Dr. Szpond concern deep classical topics in Algebra and Geometry, which received recently new impulses because of their application to combinatorics, and are considered of high interest in the modern scientific literature. The achievements of the candidate provide an important contribute to the theory, and denote an adequate level of scientific maturity. The academic activities, both from the educational and from the organizational point of view, are sufficiently wide to suggest that Dr. Szpond can produce a significant, positive impact in universitary structures.

In conclusion, my vote is in favor of the conferment of the Habilitation.

Siena, 5 Nov. 2021

(Luca Chiantini

Università di Siena, Italy)