FACULTY OF ELECTRICAL ENGINEERING
Prof. RNDr. Jan Hamhalter, CSc.
Head of the Department of Mathematics
hamhalte@fel.cvut.cz

CTU CZECH TECHNICAL UNIVERSITY IN PRAGUE

05th March 2024, Prague

Review of the scientific achievements of Dr. Hanna Podsedkowska for the habilitation committee

The scientific contribution of Dr. Hanna Podsedkowska concerns the concept of entropy in the context of von Neumann algebras. This is an important topic of the structure theory of operator algebras and quantum information theory with vital applications to quantum physics. Mainly, she has obtained far reaching results on the structural properties of Segal entropy for semifinite von Neumann algebras including important subadditivity inequalities and tensor product properties. In more details, I will focus on the content of the following research papers stated in the "Description of Achievements" in the document "Summary of Professional Achievements".

1. A.Luczak, H.Podsedkowska: *Properties of Segal's entropy for quantum systems*, International Journal of Theoretical Physics 2017 **56**: 3783-3793.
2. H.Podsedkowska: *Entropy of Quantum Measurement,* Entropy 2015, **17** (3), 1181-1196, doi: 10.3390/e17031181.
3. H.Podsedkowska: *Strong subadditivity of quantum mechanical entropy for semi-finite von Neumann algebras*, Studia Mathematica 2020.
4. A.Luczak, H.Podsedkowska, M. Seweryn: *Maximum Entropy Models for Quantum Systems* Entropy 2017 19(1) :1.
5. H.Podsedkowska, R.Wieczorek: *Holevo type bounds for general quantum system,* Reports on Mathematical Physics 2017 **80** (3) 349-360.

Let us first introduce the background. Classical concept of quantum mechanical entropy introduced by John von Neumann in 1932 has the form

$$H(\varrho) = -tr(h_\varrho \log h_\varrho)\,,$$

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where ϱ is a normal state on $B(H)$ (algebra of bounded operators on a Hilbert space H) with representing trace class nonnegative operator h_ϱ in the sense

$$\varrho(x) = tr(xh_\varrho)\,.$$

Let us now replace the algebra $B(H)$ by a semifinite von Neumann algebra M with a faithful normal semifinite trace τ. Then for any normal state $\varrho \in M_*$, there is a unique nonnegative operator h_ϱ (possibly unbounded) affiliated to M and integrable with respect to τ, such that

$$\varrho(x) = \tau(h_\varrho x) \quad \text{for all } x \in M\,.$$

Now one can define the Segal entropy $S(\varrho)$ as follows

$$S(\varrho) = -\tau(h_\varrho \log h_\varrho)\,.$$

The applicant has developed the theory of this important concept with far reaching consequences for quantum information theory. Let us remark that for doing that it was necessary to overcome many difficulties consisting in considerable difference between von Neumann and Segal entropy. In fact, in the context of von Neumann entropy on $B(H)$ the operators h_ϱ have discrete spectrum which is not true in case of general semifinite von Neumann algebras. The more striking difference causing not only technical difficulties is the fact that h_ϱ is an unbounded operator in case of Segal entropy. The work of Hanna Podsedkowska is worth in overcoming these points. Let us also note that von Neumann entropy concerns quantum mechanical systems while Segal's entropy involves also quantum field theory and quantum statistical physics, which underlines importance and applications of the work of the applicant.

Let me now briefly comment on the papers.

In the first paper [1] the lower semicontinuity of the Segal entropy on bounded parts has been established. Moreover, it was shown that for bounded density operators the Segal entropy decreases after going through a quantum channel.

In the paper [2] the subadditivity of the Segal entropy is established for a finite trace and finite densities.

TECHNICKÁ 2
166 27 PRAHA 6
CZECH REPUBLIC

+420 224 352 016
EMAIL@FEL.CVUT.CZ
WWW.FEL.CVUT.CZ

VAT CZ68407700
KB PRAHA 9, BIC KOMBCZPPXXX
IBAN CZ9401000000195504540257

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In [3] the research on strong subadditivy has been crowned by proving beautiful and very general strong subadditive inequality for tensor products of three algebras, generalizing prominent results of Araki-Lieb and Lieb-Ruskai.

In the paper [4] it is shown that the maximal value of entropy of the state with fixed energy level is attained for a Gibb's state.

In the final paper [5], among others, the quantity called accessible information is estimated. In case of the finite faithful normal trace and states with bounded density, strong and nice generalization of the famous Holevo theorem (i.e. estimation in the case of canonical trace on $B(H)$) is proved.

Conclusions

The scientific contribution of Dr. Hanna Podsedkowska meets high international level in a competitive area of operator algebras and quantum information theory with applications to quantum physics. She has solved important open problems and earned international recognition. She has ability to collaborate with other scholars and to explain her ideas to a wider audience. Based on it, it is my pleasure to strongly recommend to award the degree of a habilitation doctor to Hanna Podsedkowska.

České vysoké učení technické v Praze
FAKULTA ELEKTROTECHNICKÁ
Katedra matematiky
K 13116
Praha 6 - Dejvice

Prof. RNDr. Jan Hamhalter, CSc.
head of the Department of Mathematics
Faculty of Electrical Engineering
Czech Technical University in Prague
Technicka 2, 166 27
Prague 6, Czech Republic